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ℓ - Group on Fuzzy Singletons

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Abstract

The theory of fuzzy sets was introduced by L.A.Zadeh in 1965. Since then, the concept of fuzzy sets has been developing in variety of theoretical and application oriented fields. The notion of fuzzy singletons was introduced by Pao and Liu [11]. In this present paper, the idea of ℓ-group generated by the collection of all fuzzy singletons is studied. Some related results related to this topic are also derived.

Keywords: ℓ-group, Fuzzy Singletons.

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1 Introduction

In order to solve complicated problems in Engineering, Medical Science, Social Science, methods in classical mathematics are not always successfully because various uncertainties are typical for these problems. Therefore there has been a great deal of alternative research and applications in the literature concerning the special tool fuzzy set theory. L.A.Zadeh[16] introduced the concept of fuzzy sets in 1965. Also fuzzy group was introduced by Rosenfeld[13]. The partially ordered algebraic systems play an important role in algebra. Some important concepts in partially ordered algebraic systems are lattice ordered groups (ℓ-groups) and lattice ordered rings. These concepts play a major role in many branches of Algebra with wide ranging applications in many disciplines. This provides sufficient motivation to researchers to review various concepts and results from the realm of abstract algebra in the broader frame work of fuzzy setting. The notion of fuzzy singletons was introduced in [11]. In this paper a new notion of ℓ-groups on the collection of all fuzzy singletons is introduced and studied.

2 Preliminaries

Definition 2.1 Let X be any nonempty set and let I = [0,1]. Then the map $\mu: X \to I$ is called a fuzzy subset of X.

Definition 2.2 Let μ be a fuzzy set on a non empty set X and $t \in [0,1]$. Then the set $\mu_t = \{x \in X / \mu(x) \geq t\}$ is called the level set of μ.

Definition 2.3 Let μ be a fuzzy set on a non empty set X. Then the set $\{x \in X / \mu(x) > 0\}$ is called the support of μ and it is denoted by Supp(μ).

Definition 2.4 Let X be any non empty set. A fuzzy point p of X is a fuzzy subset of X which has singleton support {x} and fuzzy value p(x) ϵ (o,1].

Definition 2.5 Let X be any non empty set. A fuzzy singleton p of X is a fuzzy subset of X which has singleton support {x} and fuzzy value p(x) ϵ (o,1].

Definition 2.6A non-empty set G is called a lattice ordered group (ℓ - group)iff

(i) $(G,+)$ is a group

(ii) $(G,\leq)$ is a lattice

(iii) $x \leq y$ implies $a+x+b \leq a+y+b$ for all $a,b,x,y \in G$.

Definition 2.7A non-empty set G is called a lattice ordered group (ℓ - group) iff

(i) $(G,+)$ is a group

(ii) $(G,\vee,\wedge)$ is a lattice.

(iii) $a+(x \vee y)=(a+x)\vee(a+y)$ and

$a+(x \wedge y)=(a+x)\wedge(a+y)$ for all $a,b,x,y \in G$.

Result 2.8 The above two definitions of ℓ - group are equivalent.

Definition 2.9A subset H of an ℓ - group G is called an ℓ - subgroup if H is a subgroup and a sublattice in G.

Definition 2.10[13]Let G be an ℓ-group. A fuzzy set μ of G is said to be a fuzzy ℓ-group of G if

(i) $\mu(x+y)\geq \mu(x)\wedge \mu(y)$

(ii) $\mu(-x)=\mu(x)$

(iii) $\mu(x\wedge y)\geq \mu(x)\wedge \mu(y)$

(iv) $\mu(x\vee y)\geq \mu(x)\wedge \mu(y)$ for all x, y in G

Definition 2.11 If μ is a fuzzy ℓ-group of G then $\mu(0) \geq \mu(x)$ for all $x \in G$.

Result 2.12 Intersection of any family of fuzzy ℓ-groups of G is a fuzzy ℓ-group of G.

3 ℓ-group on fuzzy singletons

Definition 3.1 Let $(G, +, \wedge,\vee)$ be a ℓ-group and $\mu : G \to [0,1]$ be a fuzzy sub set of G. Then the collection of all fuzzy singletons of G forms a ℓ-group G_μ with respect to the following operations:

$(f_1 \bar{\vee} f_2)(x) = f_1(x) \vee f_2(x)$

$(f_1 \bar{\wedge} f_2)(x) = f_1(x) \wedge f_2(x)$

$(f_1 \circ f_2)(x) = \bigvee_{x=y+z} [f_1(y) \wedge f_2(z)]$ for all $x,y,z \in G$.

The system $(G_\mu, +, \bar{\vee}, \bar{\wedge})$ is called the ℓ-group generated by fuzzy singletons of G.

Proposition 3.1 Let G be a ℓ-group and G_μ be the ℓ-group generated by the fuzzy singletons of G. Then $f_e \in G_\mu$ is the identity element if $f_1 \bar{\vee} f_2 \bar{\vee} \ldots \bar{\vee} f_e \bar{\vee} \ldots = f_e$.

Proof

Assume that$f_1(x_1) \bar{\vee} f_2(x_2) \bar{\vee} \ldots \vee f_e(x_i) \bar{\vee} \ldots = f_e(x_i)$.

Now $(f_1 \circ f_e)(x) = \bigvee_{x=y+z} [f_1(y) \wedge f_e(z)]$

$= \bigvee_{x=y+z} [f_1(y)]$

$=f_1(x)$.

Similarly $(f_2 \circ f_e)(x)=f_2(x)$, $(f_3 \circ f_e)(x)=f_3(x)$ and so on.

Thus f_e is an identity element of G_μ.

Proposition 3.2 Let G be a ℓ-group and G_μ be the ℓ-group generated by the fuzzy singletons of G. Then G_μ is abelian.

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Proof

Let $G_\mu = \{ f_1, f_2, f_3 \ldots\}$ where $f_1, f_2, \ldots$ are the fuzzy singletons on G.

Now $(f_1 \circ f_2)(x) = \bigvee_{x=y+z} [f_1(y) \wedge f_2(z)]$

$= \bigvee_{x=z+y} [f_2(z) \wedge f_1(y)]$

$=(f_2 \circ f_1)(x)$

Thus G_μ is abelian.

Proposition 3.3 Let G be a ℓ-group and G_μ be the ℓ-group generated by the fuzzy singletons of G. Then G_μ is distributive.

Proof

Let $G_\mu = \{ f_1, f_2, f_3 \ldots\}$ where $f_1, f_2, \ldots$ are the fuzzy singletons on G.

$(f_1 \circ (f_2 \barwedge f_3))(x) = \bigvee_{x=y+z} [f_1(y) \wedge ((f_2 \barwedge f_3)(z))]$

$= \bigvee_{x=y+z} [(f_1(y) \wedge (f_2(z)) \wedge f_3(z))]$

$= \bigvee_{x=y+z} [(f_1(y) \wedge f_1(y)) \wedge (f_2(z)) \wedge f_3(z))]$

$= \bigvee_{x=y+z} [((f_1(y) \wedge (f_1(y) \wedge f_2(z)) \wedge f_3(z)]$

$= \bigvee_{x=y+z} [((f_1(y) \wedge (f_2(z)) \wedge f_1(y)) \wedge f_3(z)]$

$= \bigvee_{x=y+z} [(f_1(y) \wedge f_2(z)] \wedge [\bigvee_{x=y+z}[f_1(y)) \wedge f_3(z)]]$

Thus $(f_1 \circ (f_2 \barwedge f_3)) = (f_1 \circ f_2) \barwedge (f_1 \circ f_3)$.

Thus G_μ is distributive.

Definition 3.4 Let G,H be twoℓ-groups and G_μ , $H_\lambda$be the ℓ-groups generated by the fuzzy singletons of G, H respectively. The direct product of G_μ and H_λ is defined by the algebraic system $(G_\mu \times H_\lambda, \circ)$ in which the operation $\circ$ on $G_\mu \times H_\lambda$is given by

$(g_1,h_1) \circ (g_2,h_2)=(g_1 \circ g_2, h_1 \circ, h_2)$for any (g_1,h_1) , $(g_2,h_2) \in G_\mu \times H_\lambda$

Proposition 3.5 Let G,H be two ℓ-groups and G_μ , $H_\lambda$be the ℓ-groups generated by the fuzzy singletons of G, H respectively. Then the direct product $(G_\mu \times H_\lambda, \circ)$ is a group.

Proof

Let(g_1,h_1) , (g_2,h_2) ,$(g_3,h_3) \in G_\mu \times H_\lambda$.

Now $(g_1,h_1) \circ ((g_2,h_2) \circ (g_3,h_3)) = (g_1,h_1) \circ ((g_2 \circ g_3 , h_2 \circ h_3))$

$=(g_1 \circ (g_2 \circ g_3) , h_1 \circ (h_2 \circ h_3))$

$=(g_1 \circ g_2) \circ g_3 , (h_1 \circ h_2) \circ h_3$

$=(g_1 \circ g_2 , h_1 \circ h_2) \circ (g_3,h_3)$

$= ((g_1,h_1) \circ (g_2,h_2)) \circ (g_3,h_3)$

$(g_1,h_1) \circ ((g_2,h_2) \circ (g_3,h_3)) = ((g_1,h_1) \circ (g_2,h_2)) \circ (g_3,h_3)$ *(Associative Property)*

Consider the identity elements $I_\mu \in G_\mu$ and $I_\lambda \in H_\lambda$.

Now$(g_1,h_1) \circ (I_\mu ,I_\lambda) = (g_1 \circ I_\mu , h_1 \circ I_\lambda)$

$= (g_1,h_1)$ since$I_\mu$andI_λ are the identity elements of G_μ and H_λ.

$\Rightarrow (I_\mu ,I_\lambda)$ is the *identity element* of $(G_\mu \times H_\lambda, \circ)$.

Hence $(G_\mu \times H_\lambda, \circ)$ is a group.

Conclusion

In this paper, the definition and properties on l-group generated by fuzzy singletons are given. In future, the concept of ideals and filters of this l-group can be developed. Using these, various application oriented results can be developed.

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Data Availability

The data used to support the findings of this study are from other scholarly papers for the simulation. The papers are properly cited in this article.

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